A Macroscopic Traffic Data Assimilation Framework Based on Fourier-Galerkin Method and Minimax Estimation

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Abstract— **In this paper, we propose a new framework for macroscopic traffic state estimation based on the Fourier-Galerkin projection method and minimax state estimation approach. We assign a Fourier-Galerkin reduced model to a partial differential equation describing a macroscopic model of traffic flow. Taking into account a priori estimates for the projection error, we apply the minimax method to construct the state estimate for the reduced model that gives us, in turn, the estimate of the Fourier-Galerkin coefficients associated with a solution if the original macroscopic model. We illustrate our approach with a numerical example.**

I. INTRODUCTION

Macroscopic models of traffic flow can be used together with traffic data in order to estimate the state of traffic on a link or a set of links. Macroscopic models are mathematically described by Partial Differential Equations (PDEs). The basic idea is to run the model, which outputs a solution in time and space, and to obtain an estimate of the state by combining this solution with the available data over the spatio-temporal domain. Past approaches have involved linearizing [1] the model in order to apply Kalman filtering [2], [3], or using the nonlinear model as it stands, and employing filtering techniques such as Extended Kalman Filtering [4] or Particle Filtering [5]. While each category of methods has its respective advantages, what they have in common is that they are applied to a reduced model which is obtained from the original macroscopic model by 'local' methods. Local methods use limited information from the spatial domain in order to compute the solution at a given point in that domain. Specifically, all of the approaches listed above use finite differences as a solution method for the underlying PDE, where the solution in a spatial "cell" is influenced by the solution in its neighbouring cells. The resulting system is a set of difference equations, each describing the evolution of the state variable (typically density) at a given point in space.

In contrast to local solution methods, 'global' methods use information from the entire spatial domain and deliver continuous (in space) approximations for the solution to the macroscopic model. This allows the solution at any point in the domain to be computed. While local methods can capture shocks, and provide an adequate solution to the model, they have certain drawbacks when it comes to data-assimilation. Specifically, the discretization in space should take into account locations of the sensors in order to make it possible to relate the state of the reduced model and the measurements. This becomes a problem if sensors are moving or their locations are given imprecisely.

In this paper, we propose a global approach to traffic state estimation by using a global method, namely the Fourier-Galerkin method, to transform the macroscopic traffic model to a system of ordinary differential equations, and then perform the data assimilation on the resulting system using minimax estimation. Fourier-Galerkin method belongs to the class of so-called projection methods. It suggests to project a solution of the macroscopic model onto a subspace generated by $\{e^{inx}\}_{|n|\le N/2}$ and then describe the evolution of the projection coefficients only. We refer the reader to [6] for further details on spectral projection methods.

In order to capture shock formation and correctly track shocks we apply a modification of the vanishing viscosity method that allows us to efficiently deal with Gibbs phenomenon (see [7]). Fourier-Galerkin method provides estimates of the projection error in L^2-norm. We stress that there is no statistical information available on projection errors and therefore statistical uncertainty description of Kalman filter or particle filters does not apply. This motivated us to apply the minimax approach. It allows to construct a guaranteed estimate of the state of a differential equation with uncertain but bounded initial condition and model errors (in our case these errors are associated with a projection error which is bounded in L^2). Minimax approach is based upon the following idea: to minimize the worst-case estimation error. In other words, one computes the estimation error for the worst-case realization of uncertain model error and initial condition and then defines the minimax estimate as such that has a minimal worst-case error. This shows that the minimax estimate is robust to all possible realizations of uncertain parameters lying within the given bounding set. We refer the reader to [8], [9], [10] and [11] for the basic information on the minimax framework. Discussion on robust projection methods by means of the minimax state estimation approach can be found in [12], [13], [14].

Our iterative state estimation approach is based on the following idea: we consider a given estimate of the state and use the bilinear structure of the reduced model corresponding to the conservation law in order to generate a new estimate. Each iteration of the process is based on the previous estimate and is obtained applying the minimax approach to the linear state equation with a parameter. One stage of the proposed procedure can be seen as a fixed-point iteration: given the estimate on the previous iteration we construct a linear system and estimate its state in order to produce a minimax state estimate which will be used in the

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next iteration as a starting point. A similar algorithm was proposed for image motion estimation in [15]. Although we do not present a rigorous convergence proof, the numerical results look promising.

The remainder of the paper is organized as follows: in Section II, we describe the flow model used in the work, and in Section III, we explain briefly how finite-difference methods have previously been used to obtain a system for state estimation, and motivate the use of global solution methods. In Section IV, we describe the Fourier-Galerkin method and use it to derive a reduced model for the traffic PDE. In Section V, we outline the data-assimilation procedure for the reduced model. Numerical results are presented in Section VI where and we explain advantages of our method over the approaches based on local methods. We describe future work in Section VII.

II. FLOW MODEL

We employ the Lighthill-Whitham-Richards [16], [17] (LWR) model. This is the standard equilibrium traffic flow model consisting of a scalar conservation law,

$$u(x,t)_t + f(u(x,t))_x = 0 \tag{1}$$

with initial data

$$u_0(x) = u(x,0) \tag{2}$$

where $u : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ is the traffic density, $x \in \mathbb{R}$ and $t \in \mathbb{R}_+$ are the independent variables, space and time respectively, and $f : \mathbb{R} \to \mathbb{R}$ is the flux function. A typical flux function is that of Greenshields [18], given by

$$f(u) = uV_m\left(1 - \frac{u}{u_m}\right) \tag{3}$$

where the constants V_m and u_m are the maximum speed and the maximum density respectively.

III. FINITE-DIFFERENCE-BASED STATE ESTIMATION

A standard way of solving this model is to use finite differences. Using that approach, the density solution obtained is a grid function. That is, at each computational time-step, Δt, the solution given by the finite difference method is

$$u_i^j = u(i\Delta x, j\Delta t),\ i = \ldots -1, 0, 1\ldots;\ j = 0, 1, 2, \ldots \tag{4}$$

Defining the vector $U^j = u_i^j$, $i = \ldots - 1, 0, 1\ldots$, an n-level difference scheme is one for which the solution U^j is constructed from $U^{j-1}, U^{j-2}, \ldots, U^{j-n+1}$. Difference schemes which have been used to solve (1)–(2) are typically two-level methods, where the solution at a given time-step is constructed only from the solution at the previous time-step. Also, the schemes applied to this problem usually require limited information from the spatial domain. For a first-order Godunov scheme applied to a scalar conservation law, the solution u_i^j depends on either u_{i+1}^{j-1} or u_{i-1}^{j-1}, depending on the signal speed. The use of limited information is why such schemes are termed "local methods". For a two-level method, the scheme can be written as

$$U^{j+1} = \mathcal{H}U^j, \tag{5}$$

where $\mathcal{H}$ is the difference operator. For traffic flow, the signal speed, $f'(u)$, will change in sign depending on the density, u, so the difference operator will need to change the direction of spatial differencing as appropriate. As a result, $\mathcal{H}$ will depend on the grid-function U^j, meaning (5) is not a linear system. However, attempts have been made to linearize such operators. Muñoz et al. [1] took such an approach, and used a piecewise linear flux function to obtain a switched-linear system. The number of linear systems they switch between depends on some assumptions made about the number of congested and free-flow regions which can exist on a single link. This linearized model was used for estimation and control [2], [3] with the assumption that at most one of each region (congested/free-flow) can exist on a link. This resulted in a system which switched between five different linear systems, for which they used a Mixture Kalman Filter for the estimation/control. The measurements in that work are of the state (with some additive noise), so the transformation from observation space to state space is trivial; in the case where all components of the state vector, U^j can be measured, the observation operator is the identity matrix. In general, though, sensors are sparse, and the observation space is of lower dimension than the state space. The observations must be taken at grid-points, meaning that the such an approach works best for fixed sensors, and the spatial discretization should be chosen such that the grid-points coincide with the sensor locations.

Another well-known work on traffic data assimilation is that by Herrera and Bayen [19], who used a 'Newtonian relaxation method', which is a heuristic method that relaxes the dynamical model towards the observations. That scheme is suitable for mobile sensors through the use of an additional term in the flow model, which acts as either a source or a sink, in order to relax the conservation law towards the measurements from GPS-equipped vehicles. This method also falls into the 'local' category because if its use of finite differences to discretize the model. Therefore, the incorporation of the measurements into the model is affected by the choice of spatial discretization.

In the following section, we describe spectral (global) solution methods, and will later show how using the resulting system for estimation allows us to take measurements from any point in space, unlike in finite-difference-based methods.

IV. GALERKIN METHOD FOR LWR MODEL

We consider the initial value problem defined by (1) and (2), where $x \in [0, 2\pi]$ and $t \geq 0$. We also restrict our attention to periodic boundary conditions, $u(0,t) = u(2\pi, t)$. In the Fourier-Galerkin method, solutions are sought in the space, span$\{e^{inx}\}_{|n|\leq N/2}$ [6]:

$$u_N(x,t) = \sum_{n=-N/2}^{N/2} a_n(t)e^{inx}. \tag{6}$$

Defining the residual,

$$R_N(x,t) = \frac{\partial u_N}{\partial t} + \frac{\partial f(u_N)}{\partial x} \tag{7}$$

and requiring it to be orthogonal to span$\{e^{inx}\}_{|n|\le N/2}$, we obtain the coefficients $a_n(t)$. However, solutions to (1) can develop shock-discontinuities, even for smooth initial data. This will give rise to strong oscillations which will spread to the entire spatial domain, causing the Fourier-Galerkin method to fail. Tadmor [7] showed that the addition of "spectral viscosity" to a scalar conservation law can help overcome this problem, and allow, moreover, the correct entropy solution to be obtained. The conservation law with viscosity is

$$u_t + f(u)_x = \varepsilon u_{xx}, \tag{8}$$

where ε is a constant. Substituting (6) into (8), and using the flux function (3), where we set $V_m = u_m = 1$, leads to the residual,

$$\begin{aligned} R_N(x,t) = &\sum_{n=-N/2}^{N/2} \dot{a}_n(t)e^{inx} + \sum_{n=-N/2}^{N/2} ina_n(t)e^{inx} \\ &- 2\sum_{k-N/2}^{N/2}\sum_{n=-N/2+k}^{N/2+k} ika_{n-k}(t)a_k(t)e^{inx} \\ &+ \varepsilon \sum_{n=-N/2}^{N/2} n^2 a_n(t)e^{inx}. \end{aligned} \tag{9}$$

Forcing the orthogonal projection of this residual onto span$\{e^{inx}\}_{|n|\le N/2}$ to vanish, we obtain the following system of ODEs,

$$\begin{aligned} \frac{da_n}{dt} = &\sum_{\substack{k=-N/2 \\ |n-k|\le N/2}}^{N/2} 2ika_{n-k}(t)a_k(t) - ina_n(t) - \varepsilon n^2 a_n(t), \\ &n = -N/2 \ldots N/2. \end{aligned} \tag{10}$$

The last term on the right hand side is due to the viscosity. Tadmor [7] showed that for scalar conservation laws, the addition of numerical viscosity on the higher modes leads to convergence to the unique entropy solution. Thus, the viscosity term is only activated for $|n/2| \ge m$, where m is some threshold wave number. This will be discussed in further detail in Section VI. The initial coefficients can be obtained from the data (2) by computing the integral,

$$a_n(0) = \frac{1}{2\pi}\int_0^{2\pi} u_0(x)e^{-inx}dx, \quad n = -N/2 \ldots N/2. \tag{11}$$

In order to proceed with the data assimilation, we now express the system as a differential equation, and define an observation operator. We write (10) as

$$\frac{dX}{dt} = A(X)X, \tag{12}$$

where $X : \mathbb{R} \to \mathbb{C}^{N+1}$ is a vector containing the $N+1$ expansion coefficients, and $A : \mathbb{C}^{N+1} \to \mathbb{C}^{N+1}$ is constructed using (10). The observation equation is given by

$$Y(t) = HX(t) + \eta(t), \tag{13}$$

where $Y : \mathbb{R} \to \mathbb{R}^M$ is a vector of density measurements, and $H : \mathbb{C}^{N+1} \to \mathbb{R}^M$ is the observation operator, which is a mapping from the complex state space of expansion coefficients to the real observation space of measured densities. This operator is constructed using (6), and the M density measurements can be taken anywhere on the interval, $x \in [0, 2\pi]$.

In the following section, we will use the system (12)–(13) for data assimilation.

V. DATA ASSIMILATION

In this section we describe the experimental data assimilation algorithm used in this paper. It is based on the minimax state estimation framework. We briefly recall it here. Assume that the state of the process of interest (a vector of projection coefficients in our case) at time instant t is described by the vector $X(t)$ and the evolution of $X(t)$ over time is governed by a system of differential equations:

$$\frac{dX}{dt} = A(p)X + e^m(t), \quad X(0) = X_0 + e^b(0)\,, \tag{14}$$

where e^m represents model error and e^b describes the error in the initial condition, p is a parameter, possibly depending on the state vector X. In our case the term e^m addresses the projection error introduced by the Galerkin method. In other words, e^m models an impact of higher order Fourier modes onto those described by the Galerkin model (12). Since $u(x,t)$ is at least square-integrable in space for every t we can assert that:

1) the vector of projection coefficients $X(t)$ is bounded in L^2;
2) the error in the initial condition e^b is bounded in $\mathbb{R}^n$.

One can prove (we refer the reader to [6, p.251] for further details on estimating the projection error) using properties of the conservation law (8) that the projection error e^m is bounded in L^2 together with initial condition. Now, assuming that the vector of density observations $Y(t)$ is related to the vector of projection coefficients $X(t)$ through the relation (13) and the observation error η is bounded we propose the following uncertainty description:

$$\|Q_0 e^b\|^2_{\mathbb{R}^n} + \int_0^T \|Qe^m\|^2_{\mathbb{R}^n} + \|R^{-1}\eta\|^2_{\mathbb{R}^m} dt \le 1\,, \tag{15}$$

where Q_0, Q, R are symmetric positive definite matrices representing worst-case bounds on the energy of uncertain parameters. Now we are ready to state equations for the minimax estimate $\widehat{X}$ and minimax gain K for the linear equation (14) with a parameter:

$$\begin{aligned} \frac{dK}{dt} &= A(p)K + KA'(p) + Q^{-1} \\ &\quad - KH'R^{-1}HK, \quad K(0) = Q_0^{-1} \\ \frac{d\widehat{X}}{dt} &= A(p)\widehat{X} + K^{-1}H'R^{-1}(Y(t) - H\widehat{X}(t)), \\ &\quad X(0) = X_0\,. \end{aligned} \tag{16}$$

We note that for any solution to (14) the following estimate holds: $\|X(t) - \widehat{X}(t)\|^2_{\mathbb{R}^n} \le \|K(t)\|$, where $\|K(t)\|$ denotes the maximal eigenvalue of K.

Now, we use the following algorithm to estimate the state of the non-linear equation (12). Assume that $\widehat{X}_1$ represents an estimate of the projection coefficients X (obtained, for instance, solving the conservation law (1) for given initial and boundary conditions). We set $p = \widehat{X}_1$ in (14) and denote by $\widehat{X}_2$ the minimax estimate obtained from (16) with $p = \widehat{X}_1$. In such a way we get a mapping of a given estimate X_1 into the corresponding minimax estimate $\widehat{X}_2$. Now, thinking of this mapping as a contraction we approximate its fixed point, that is we assign to the minimax estimate $p = \widehat{X}_2$ a new minimax estimate $\widehat{X}_3$ and repeat this procedure until the convergence is achieved. This gives us a sequence of estimates $\{\widehat{X}_k\}$. Assuming that this sequence is convergent we can define the estimate of the non-linear equation (12) as a limiting point of $\{\widehat{X}_k\}$. The following section shows the numerical results on the convergence of the proposed iterative estimation procedure.

VI. NUMERICAL IMPLEMENTATION AND RESULTS

Is this section, we present some numerical results for the Fourier-Galerkin reduced model and minimax data assimilation, demonstrating the shock capturing capabilities of the model.

A. Solution of Fourier-Galerkin reduced model

We integrate the system (10) in time using a 4-th order Runge-Kutta scheme (RK4). Note that RK4 does not conserve the integral of the density over the domain and this method is, therefore, non-suitable for long-term simulations. We first get the initial coefficients by computing the integral (11). This can be done by quadratures or by using Fast Fourier Transform (FFT). We start with the initial data,

$$u_0(x) = e^{-20(x-1)^2}, \tag{17}$$

which is a Gaussian, centred at $x = 1$. The viscosity term in (10) is only activated for the higher modes [7], $|n/2| \ge m$, where m is the wave number beyond which the viscosity is added. This can be achieved by replacing the viscosity term with $-H[|n/2| - m]\varepsilon n^2 a_n(t)$, where $H[\cdot]$ is the heaviside step function. Tadmor [7] solved burgers' equation using $\varepsilon m \approx 0.25$. Having obtained the $N + 1$ initial coefficients, $a_n(0)$, for the initial data, (17), we solve the system, (10), using RK4 with $N = 100$ and $m = 30$.

Fig. 1 shows the solution at different times of the initial value problem with periodic boundary conditions. Also shown is the solution to the same problem using a first-order Godunov scheme. Fig. 1a shows the smooth initial profile, and Fig. 1b shows the profile steepening at $t = 0.1$, and a shock beginning to form. We see that oscillations start to form here. In Fig. 1c, we see that at $t = 1$, the solution is a shock wave and a rarefaction wave, and that oscillations are still present. However, the solution is still stable. Fig. 1d shows the solution at $t = 5.7$. The oscillations have been damped out, except near the shock. However, since the Fourier-Galerkin solution still coincides with the Godunov solution, the former appears to capture shocks successfully. The inclusion of periodic boundary conditions is evident from the last figure.

B. Data assimilation results

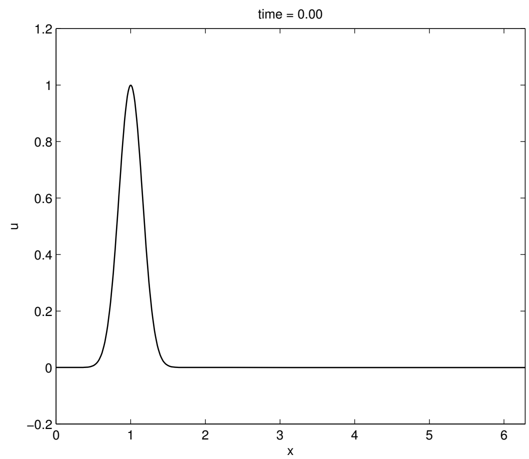
To perform the data assimilation, we run the model with initial data (17) until $t = 5.7$, and use the Godunov solution of the initial value problem, (1) and (17), to generate the measurements. Then we perturb the measurements artificially with additive noise which has error covariance matrix $R = 100Id$ (so we allow quite large noise in observations), Id stands for identity matrix. In other words, we use the Godunov solution as "ground truth", sampling values of its solution and assimilating them into the Fourier-Galerkin model. We assume that we can take M measurements at points $\{x_1, x_2, \dots, x_M\}$, where $0 < x_1 < x_2 < \dots < x_M \le 2\pi$, and we choose the Godunov spatial discretization in such a way that it coincides with the locations of the M observations. At each time-step of RK4, we construct the operators, A and H, and we use (16) for the estimation with $Q_0 = 10Id$, $Q = Id$ (this shows that we trust our Fourier-Galerkin model). Fig 2 shows the results after few iterations of the algorithm presented in Section V. Also shown are the perturbed and unperturbed observations. We observe a good match between the estimate and the measurements, which demonstrates the numerical convergence of the iteration process for the chosen case. We note that the iteration process might fail if the initial guess was far off the measurements. This point is left for future research.

An important property of the observation operator, H, is that it can be constructed from (6) in such a way as to map from the state space to an observation space in which the measurements are taken at any points on the interval $x \in [0, 2\pi]$. This is in contrast to finite difference-based methods, which require measurements from fixed points in space, which ideally should coincide with the grid-points of the spatial discretization.

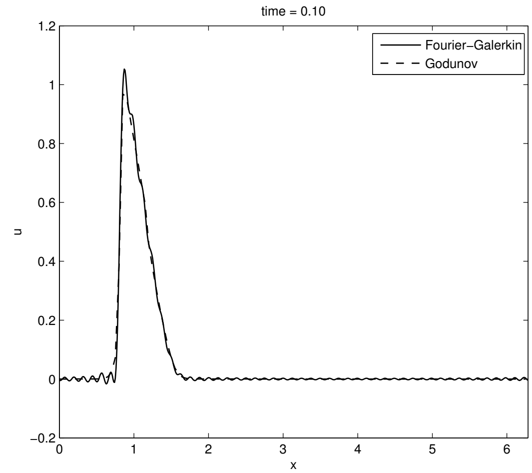
VII. CONCLUSIONS AND FUTURE WORK

We distinguish between finite-difference-based traffic state estimation, and estimation based on global methods, such as the Fourier-Galerkin projection method presented here. The approach looks promising. The continuous-in-space reconstruction does not require the observation locations to coincide with grid-points, giving the method the ability to take measurements from any part of the spatial domain. This would appear to give the method a considerable advantage over finite-difference-based methods, in which measurements must be taken at grid-points.

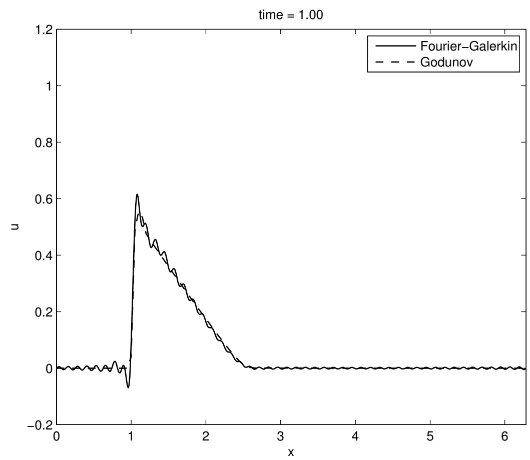
One of the main points for the future research is to address non-periodic boundary conditions that makes the reduced model much more attractive for practical applications. Another important direction is to study sufficient conditions for the convergence of the proposed iterative algorithm.



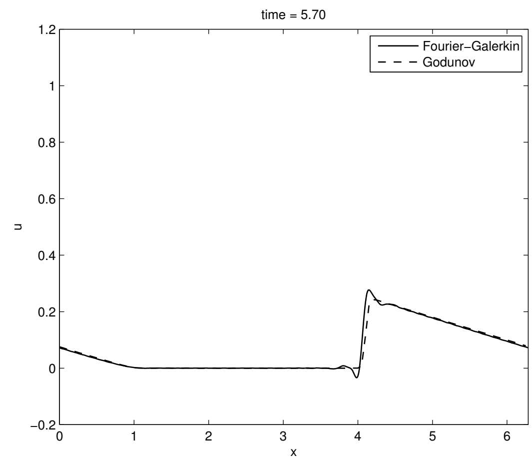
(a) Initial profile



(b) Shock forming at $t = 0.1$



(c) Shock-rarefaction solution at $t = 1$



(d) Solution at $t = 5.7$ showing periodic B.C.

Fig. 1: Numerical solution to LWR with initial data (17)

Technically it would be desirable to use an energy preserving time integrator for the reduced model.

VIII. ACKNOWLEDGMENTS

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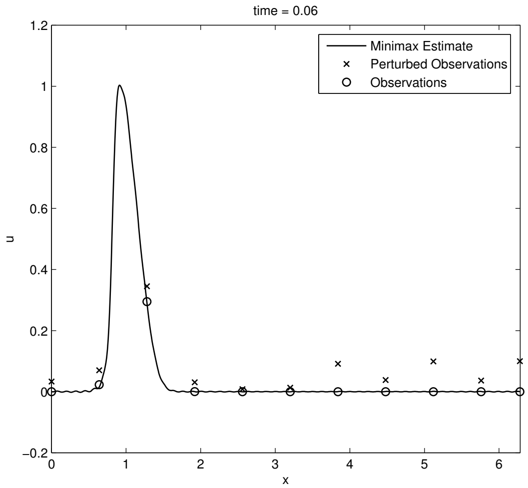
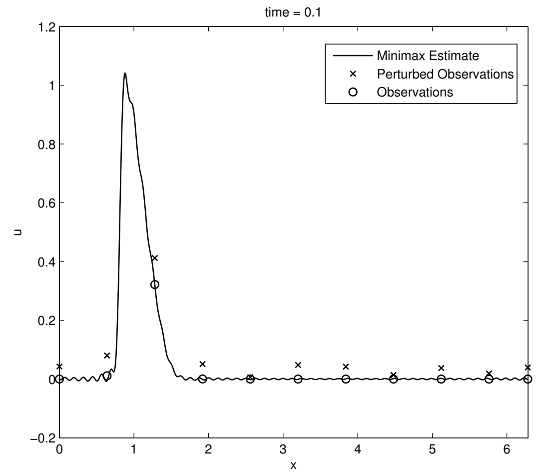
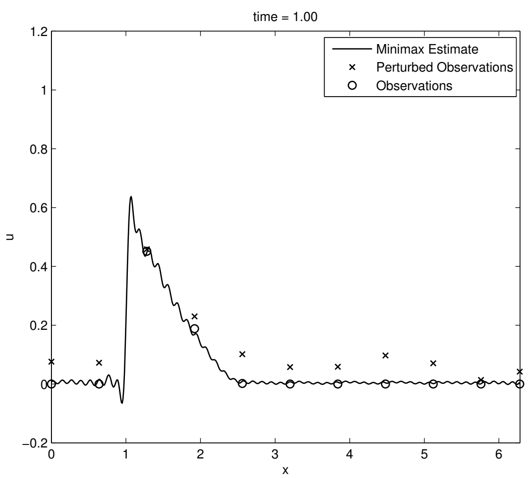


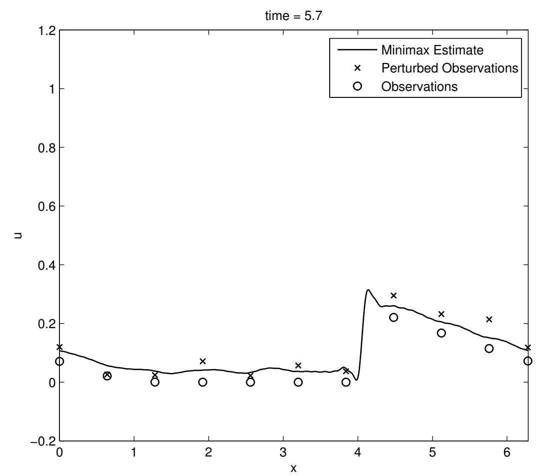
(a) Estimate and observations at $t = 0.06$



(b) Estimate: shock formation at $t = 0.1$



(c) Estimate: Shock-rarefaction solution at $t = 1$



(d) Estimate at $t = 5.7$

Fig. 2: Data assimilation results

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