Appeared in Amer. Math. Soc. Notices 21, 2, 1974, p. A-322

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$L_{\kappa,\lambda}$-equivalence of ordinals and a compactness result.

Throughout $\kappa \geq \lambda \geq \omega$, κ regular; α, β, γ are ordinals. *Definitions.* 1. Write α uniquely as $\gamma\alpha_1 + \alpha_2$, $\alpha_2 < \gamma$. Then $r_\gamma(\alpha)$ and $c_\gamma(\alpha)$ are defined to be α_2 and cofinality $(\gamma\alpha_1)$, respectively. Further, $\alpha \sim \beta$ mod γ, if (1) $r_\gamma(\alpha) = r_\gamma(\beta)$, and (2) $\alpha < \gamma$ iff $\beta < \gamma$. 2. $\mathfrak{A} \equiv \mathfrak{B}$ means that $\mathfrak{A}$ and $\mathfrak{B}$ agree on $L_{\kappa,\lambda}$-sentences. *Theorem.* 1. If λ is a successor or ω, $\kappa = \lambda$, then $<\alpha; \epsilon> \equiv_{\kappa,\lambda} <\beta; \epsilon>$ iff (1) $\alpha \sim \beta$ mod κ^κ, and (2) $c_{\kappa^\kappa}(\alpha)$ and $c_{\kappa^\kappa}(\beta)$ are equal, or both at least λ. 2. If λ is regular, $\kappa > \lambda$, then $<\alpha; \epsilon> \equiv_{\kappa,\lambda} <\beta; \epsilon>$ iff (1) $\alpha \sim \beta$ mod κ, and (2) $c_\kappa(\alpha)$ and $c_\kappa(\beta)$ are equal, or both at least λ. 3. If λ is weakly inaccessible, $\kappa = \lambda$, then $<\alpha; \epsilon> \equiv_{\kappa,\lambda} <\beta; \epsilon>$ iff (1) $\alpha \sim \beta$ mod κ, and (2) $c_\kappa(\alpha)$ and $c_\kappa(\beta)$ are equal, or both at least λ. 4. If λ is singular, $\kappa = \lambda^+$, then $<\alpha; \epsilon> \equiv_{\kappa,\lambda} <\beta; \epsilon>$ iff (1) $\alpha \sim \beta$ mod κ^κ, and (2) $c_{\kappa^\kappa}(\alpha)$ and $c_{\kappa^\kappa}(\beta)$ are equal, or both at least λ^+. 5. If λ is singular, $\kappa > \lambda^+$, then $<\alpha; \epsilon> \equiv_{\kappa,\lambda} <\beta; \epsilon>$ iff (1) $\alpha \sim \beta$ mod κ, and (2) $c_\kappa(\alpha)$ and $c_\kappa(\beta)$ are equal, or both at least λ^+. □ The theorem yields a complete characterization of ordinals in $L_{\kappa\lambda}$ (using only $<$ and $=$), from which follows *Corollary.* Assume $\kappa > \lambda$, or κ is weakly inaccessible; $\kappa = \lambda$. Let Σ be a set of $L_{\kappa\lambda}$ sentences involving only $<$ and $=$, $\ni$ every subset of Σ having less than κ sentences has a well-ordered model. Then Σ has a well-ordered model. (Received January 7, 1974.)