CORRIGENDUM
Reasoning about Knowledge and Probability

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Please note the following corrections to "Reasoning About Knowledge and Probability" by Ronald Fagin and Joseph Y. Halpern, which appeared in Volume 41, Number 2 (March 1994) of *Journal of the ACM*, pp. 340–367.

There is a typo in the paper that recurs frequently. On page 355, lines 1, 2, 5, and 11, and on page 358, lines 8, 10, 11, 35, and 37, we write $\mu_i(\psi)$, $\mu_i(\varphi_s)$, and $\mu_i(\varphi_{s'})$; the μ_i should be replaced by w_i in each case.

More significantly, on page 353, line 28, Axiom W4, which says "$w_i(\varphi) = w_i(\psi)$ if $\varphi \Leftrightarrow \psi$ is a propositional tautology", should be replaced by an inference rule that says "From $\varphi \Leftrightarrow \psi$ infer $w_i(\varphi) = w_i(\psi)$."

The need for this inference rule (which is clearly at least as strong as the axiom) arises in the proof of Theorem 4.1, in the last line of the main text on page 354. There it is claimed that $AX_{MEAS} \vdash \psi \Leftrightarrow \bigvee_{\{s \in S | \psi \in s\}} \varphi_s$, for all $\psi \in Sub^+(\varphi)$, and that the proof uses only propositional reasoning, namely K1 and R1. While it is true that $AX_{MEAS} \vdash \psi \Leftrightarrow \bigvee_{\{s \in S | \psi \in s\}} \varphi_s$, it is not true that this can be shown using only propositional reasoning. For example, let φ be $K_1 p$, so that $\mathrm{Sub}^+(\varphi) = \{p, \neg p, K_1 p, \neg K_1 p\}$. Then S consists of three maximal consistent subsets s_1, s_2, s_3 of $\mathrm{Sub}^+(\varphi)$, with (a) $\varphi_{s_1} = p \wedge K_1 p$, (b) $\varphi_{s_2} = \neg p \wedge \neg K_1 p$, and (c) $\varphi_{s_3} = p \wedge \neg K_1 p$. Let ψ be $K_1 p$. Then $\bigvee_{\{s \in S | \psi \in s\}} \varphi_s$ is simply φ_{s_1}, that is, $p \wedge K_1 p$, which is indeed provably equivalent to ψ, that is, $K_1 p$. But to prove this requires K3. Because more than propositional reasoning is needed here, we need the stronger version of W4 to show that $w_i(\psi) = \Sigma_{\{s \in S | \psi \in S\}} w_i(\varphi_s)$ is provable in AX_{MEAS} (page 355, lines 1 and 2, with μ_i replaced by w_i, as noted above).