Shopbot Economics

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Abstract. Shopbots are Internet agents that search for information that pertains to the price and quality of goods and services. With the advent of shopbots, a dramatic reduction in search costs seems imminent, which promises (or threatens) to radically alter market behavior. This research includes the proposal and theoretical analysis of a simple economic model that is intended to capture some of the essence of shopbots, and attempts to shed light on their potential impact on market behavior. In addition, experimental simulations of an economy of software agents are described, which are designed to further study the dynamics of interaction among electronic buyers, sellers, and shopbots.

The desire to explore the economic impact of shopbots as sources of price and product information leads to a model that is similar in spirit to those that have been investigated by economists interested in understanding the phenomenon of price dispersion. The underlying assumptions and methodology of this research, however, are directed toward the eventual goal of designing economic software agents, and thus differ substantially from previous economic studies. A series of shopbot experiments are discussed in which (i) search costs are nonlinear, (ii) some portion of the buyer population does not have access to search mechanisms, and (iii) shopbots are economically motivated agents that strategically price their information services in an effort to maximize their own profits. One of the main contributions of this paper is that it explores some of the ways in which economies of the Internet — particularly, that which is conducted via software agents — vastly differs from traditional economics.

1 Introduction

Shopbots — software agents that automatically query multiple on-line vendors to gather information about prices and other attributes of consumer goods and services — herald a future in which autonomous agents become an essential component of nearly every facet of electronic commerce [3,12,18,6]. Since the launch of BargainFinder [13], a CD shopbot, on June 30, 1995, scores of shopbots have emerged which, in response to a consumer’s expressed interest in an item, query several dozen producers’ web sites, and then return sorted information to the consumer, within seconds. For example, a shopbot available at shopper.com claims to compare over 1,000,000 prices on 100,000 computer-oriented products.
Another shopbot, DealPilot.com, gathers, collates, and sorts the prices and expected delivery times of books, CDs, and movies offered for sale on-line. One of the more popular shopbots, mySimon.com, provides information about office supplies, groceries, toys, apparel, and consumer electronics, just to name a few of the items on its product line. Shopbots outperform and out-inform humans by providing extensive product coverage in just a few seconds, far more than a patient, determined human shopper could achieve after hours of manual search.

Shopbots deliver on one of the great promises of the Internet and e-commerce: a radical reduction in the cost of obtaining and distributing information. It is generally recognized that the freer flow of information will profoundly affect market efficiency, as economic friction will be reduced significantly [1, 7, 14, 4]. Transportation costs, menu costs — the costs to firms of evaluating, updating, and advertising prices — and search costs — the costs to consumers of seeking out optimal price and quality — will all decrease, as a consequence of the digital nature of information as well as the presence of autonomous agents that find, process, collate, and disseminate that information at little cost. What are the implications of the widespread use of shopbots? Specifically, do shopbots have the potential to increase social welfare? If so, how can shopbots adequately price their services so as to provide consumers with incentives to subscribe, while retaining profitability? More generally, what is the expected impact of agent technology on the nascent information economy?

Previous work in economics on the impact of search costs on equilibrium prices was oriented towards explaining the phenomenon of price dispersion in social economies; see, for example, [16, 19, 2]. In such work, an attempt is made to approximate human behavior with mathematical functions or algorithms, and under the relevant assumptions, collective behavior and equilibria are studied. In contrast with previous intentions, our mission is to investigate the possible dynamics of the future information economy in which software agents, rather than human constituents, play the key role. Consequently, we take mathematical functions and algorithms a good deal more seriously, by regarding them as precise specifications of the behavior of economic players. In this paper, we focus on the likely effect that one particular specification of a class of agents, namely shopbots, will have on electronic markets. From this study, we hope to gain insights into the design of adaptive algorithms for economically-motivated, computational agents which successfully maximize utility.

This paper is organized as follows. The next section presents our model of a simple market in which shopbots provide price information; this model is then analyzed from a game-theoretic point of view in Sec. 3.1. In Sec. 4, we consider the dynamics of interaction among software agents designed to model electronic buyers and sellers; moreover, we investigate the effect of non-linear search costs (Sec. 4.1) and irrational consumers (Sec. 4.2) via experimental simulations. In Sec. 5 we extend our model by introducing shopbots, who act as intermediaries, strategically pricing the information services they provide. Finally, in Secs. 6 and 7 we describe related and future work, respectively.
Fig. 1. Shopbot Model. Buyers' numerical labels indicate the number of price quotes compared before selecting a seller from which to potentially make a purchase.

2 Model

We consider an economy in which there is a single commodity that is offered for sale by $S$ sellers and of interest to $B$ buyers (see Figure 1). Periodically, at a rate $\rho_b$, a buyer $b$ attempts to purchase a unit of the commodity. Each attempted purchase proceeds as follows. First, buyer $b$ conducts a search of fixed sample size $i$, which entails requesting $0 \leq i \leq S$ price quotes.\footnote{We permit a search strategy of 0 to allow buyers to opt out of the market entirely, which may be desirable if search costs are prohibitive.} A search mechanism (which could be manual or shopbot-assisted) instantly provides price quotes for $i$ randomly chosen sellers. Buyer $b$ then selects a seller $s$ whose quoted price $p_s$ is lowest among the $i$ (ties are broken randomly), and purchases the commodity from seller $s$ if and only if $p_s \leq v_b$, where $v_b$ is buyer $b$'s valuation of the commodity.

In addition to the purchase price, buyers incur search costs. The cost $c_i$ of using search strategy $i$, however, does not enter into the purchasing decision of the buyers, because buyers must commit to conducting a search before the results of the search become available. In other words, search payments are sunk costs. Instead, search costs affect the choice ($0 \leq i \leq S$) of search strategy utilized by buyers. A buyer $b$ is assumed to periodically re-evaluate its strategy at a rate $\sigma_b \leq \rho_b$, where typically, $\sigma_b \ll \rho_b$. Upon re-evaluation, the rational buyer estimates a price $\tilde{p}_j$ that it would expect to pay for the commodity if it were to abide by strategy $i$, and then selects the strategy $j$ that minimizes $\tilde{p}_j + c_j$, provided that $\tilde{p}_j + c_j \leq v_b$. If this condition is not satisfied, then $j = 0$; i.e., the rational buyer does not search or participate in the market at that time.
The buyer population at any given moment is characterized by the strategy vector \( \mathbf{w} \), in which component \( w_i \) represents the fraction of buyers employing strategy \( i \) and \( \sum_{i=0}^{S} w_i = 1 \). A seller's expected profit per unit time \( \pi_s \) depends on the strategy vector \( \mathbf{w} \), the price vector \( \mathbf{p} \) describing all sellers' prices, and the cost of production \( r_s \) for seller \( s \). In particular, \( \pi_s(\mathbf{p}, \mathbf{w}) = D_s(\mathbf{p}, \mathbf{w})(p_s - r_s) \), where \( D_s(\mathbf{p}, \mathbf{w}) \) is the rate of demand for the good produced by seller \( s \), in terms of the current price and search strategy vectors. The demand \( D_s(\mathbf{p}, \mathbf{w}) \) is the product of three terms: (i) the overall buyer rate of demand, namely \( \rho = \sum_b p_b \), (ii) the likelihood that seller \( s \) is selected as a potential seller, denoted \( h_s(\mathbf{p}, \mathbf{w}) \), and (iii) the fraction of buyers whose valuations satisfy \( u_b \geq p_s \), denoted \( g(p_s) \). Specifically, \( D_s(\mathbf{p}, \mathbf{w}) = \rho B h_s(\mathbf{p}, \mathbf{w}) g(p_s) \). Without loss of generality, we define the time scale such that \( \rho B = 1 \), and we then interpret \( \pi_s \) as seller \( s \)'s expected profit per unit sold systemwide. Now, seller \( s \)'s profits are given by:

\[
\pi_s(\mathbf{p}, \mathbf{w}) = (p_s - r_s) h_s(\mathbf{p}, \mathbf{w}) g(p_s)
\]

(1)

3 Analysis

We now present a game-theoretic analysis of our shopbot model, in which we derive the Nash equilibrium, assuming first that sellers are rational (i.e., profit maximizers), and later that buyers too are rational (i.e., utility maximizers). A Nash equilibrium is a vector of prices at which sellers maximize their individual profits, buyers maximize their individual utilities, and from which no agent has any incentive to deviate [15].

3.1 Exogenous Buyer Decisions

Initially, we focus on the strategic decision-making of the sellers, by assuming the distribution of the buyer population is fixed and exogenously determined. In this case, there are no pure strategy Nash equilibria whenever \( 0 < w_1 < 1 \); a proof of this claim is provided in Appendix A. There does, however, exist a symmetric Nash equilibrium in mixed strategies, which we derive presently.

Let \( f(p) \) denote the probability density function according to which sellers set their equilibrium prices, and let \( F(p) \) denote the corresponding cumulative distribution function. Following Varian [19], we note that in the range for which it is defined, \( F(p) \) has no mass points, since otherwise a seller could decrease its price by an arbitrarily small amount and experience a discontinuous increase in profits. Moreover, there are no gaps in the distribution, since otherwise prices would not be optimal — a seller charging a price at the low end of the gap could increase its price to fill the gap while retaining its market share, thereby increasing its profits. The cumulative distribution function \( F(p) \) is computed in terms of the quantity \( h_s(\mathbf{p}, \mathbf{w}) \).

\footnote{If \( w_1 = 1 \), then the unique Nash equilibrium is such that all sellers charge the monopoly price \( v \); if \( w_1 = 0 \), then the unique Nash equilibrium is such that all sellers charge the competitive price \( r \) (see [8, 9]).}
Recall that $h_s(p, w)$ represents the probability that buyers select seller $s$ as their potential seller. This function is expressed in terms of the probabilistic demand for seller $s$ by buyers of type $i$, namely $h_{s,i}(p)$, for $0 \leq i \leq S$. The first component $h_{s,0}(p) = 0$. Consider the next component $h_{s,1}(p)$. Buyers of type 1 select sellers at random; thus, the probability that seller $s$ is selected by such buyers is simply $h_{s,1}(p) = 1/S$. Now consider buyers of type 2. In order for seller $s$ to be selected by a buyer of type 2, $s$ must be included within the pair of sellers being sampled — which occurs with probability $(S-1)/S = 2/S$ — and $s$ must be lower in price than the other seller in the pair. Since, by the assumption of symmetry, the other seller’s price is drawn from the same distribution, this occurs with probability $1 - F(p)$.

Therefore, $h_{s,2}(p) = (2/S)[1 - F(p)]$. In general, seller $s$ is selected by a buyer of type $i$ with probability $\left(\frac{S-1}{i-1}\right)/\left(\frac{S}{i}\right) = i/S$, and seller $s$ is the lowest-priced among the $i$ sellers selected with probability $[1 - F(p)]^{i-1}$, since these are $i-1$ independent events. Thus, $h_{s,i}(p) = (i/S)[1 - F(p)]^{i-1}$, and

$$h_s(p, w) = \frac{1}{S} \sum_{i=1}^{S} iw_i[1 - F(p)]^{i-1}$$  \hspace{1cm} (2)

A Nash equilibrium in mixed strategies requires that all prices that receive positive probability yield equal payoffs, otherwise it would not be optimal to randomize. Thus, assuming $r_s = r$ for all sellers $s$, the equilibrium payoff $\pi = \pi_s(p, w) = (p - r)h_s(p, w)g(p)$, for all prices $p$. The precise value of $\pi$ can be derived by considering the maximum price that sellers are willing to charge, say $p_m$. At this price, $F(p_m) = 1$, which by Eq. 2 implies that $h_s(p, w) = w_1/S$. Identifying the expression $(p - r)g(p)$ as the profit function of a monopolist, this function attains its maximal value (the monopolist’s profit) at say $\pi_m$. Therefore, for all sellers $s$, and for all prices $p$, $\pi = (w_1/S)\pi_m$, and hence,

$$(p - r)g(p) = \frac{w_1\pi_m}{\sum_{i=1}^{S} iw_i[1 - F(p)]^{i-1}}$$  \hspace{1cm} (3)

implicitly defines $p$ and $F(p)$ in terms of one another and $g(p)$, for all $p$ such that $0 \leq p \leq 1$.

The function $g(p)$ can be expressed as $g(p) = \int_{p}^{\infty} \gamma(x)dx$, where $\gamma(x)$ is the probability density function describing the likelihood that a given buyer has valuation $x$. For example, suppose that the buyers’ valuations are uniformly distributed between 0 and $v$, with $v > 0$; then the integral yields $g(p) = 1 - p/v$, for $p \leq v$. This case was studied in Greenwald, et al. [10]. In this paper, we assume $v_b = v$ for all buyers $b$, in which case $\gamma(x)$ is the Dirac delta function $\delta(v - x)$, and the integral yields a step function $g(p) = \Theta(v - p)$ as follows:

$$\Theta(v - p) = \begin{cases} 1 & \text{if } p \leq v \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (4)

In Eq. 2, $h_s(p, w)$ is expressed as a function of seller $s$’s scalar price $p$, given that probability distribution $F(p)$ describes the other sellers’ expected prices.
For the assumed distribution of buyer valuations, the monopolist’s profit function is simply \( p - r \), for \( p \leq v \), which is maximized at price \( p_m = v \). At this price, the monopolist’s profits \( \pi_m = v - r \). Inserting these values into Eq. 3 and solving for \( p \) in terms of \( F \) yields:

\[
p(F) = r + \frac{w_1(v - r)}{\sum_{i=1}^{S} iw_i[1 - F]^{i-1}}
\]

Eq. 5 has several important implications. First of all, in a population in which there are no buyers of type 1 (i.e., \( w_1 = 0 \)) the sellers charge the production cost \( c \) and earn zero profits; this is the traditional Bertrand equilibrium. On the other hand, if the population consists of just two buyer types, 1 and some \( i \neq 1 \), then it is possible to invert \( p(F) \) to obtain:

\[
F(p) = 1 - \left( \frac{w_1}{iw_i} \right) \left( \frac{v - p}{p - r} \right)^{1/i-1}
\]

The case in which \( i = S \) was studied previously by Varian [19]; in this model, buyers either choose a single seller at random (type 1) or search all sellers and choose the lowest-priced among all sellers (type \( S \)).

Since \( F(p) \) is a cumulative probability distribution, it is only valid in the domain for which its valuation is between 0 and 1. The upper boundary is \( p = p_m \), since prices above this threshold lead to decreases in market share that exceed the benefits of increased profits per unit. The lower boundary \( p^* \) can be computed by setting \( F(p^*) = 0 \) in Eq. 3, which yields:

\[
p^* = r + \frac{w_1(v - r)}{\sum_{i=1}^{S} iw_i}
\]

In general, Eq. 5 cannot be inverted to obtain an analytic expression for \( F(p) \). It is possible, however, to plot \( F(p) \) without resorting to numerical root finding techniques. We use Eq. 5 to evaluate \( p \) at equally spaced intervals in \( F \in [0,1] \); this produces unequally spaced values of \( p \) ranging from \( p^* \) to \( v \).

We now consider the probability density function \( f(p) \). For the given choice of \( g(p) \), the sellers’ profits \( \pi \) are as follows:

\[
(p - r)h_s(p, w) = (w_1 / S)(v - r)
\]

Differentiating both sides of Eq. 8 with respect to \( p \) and substituting Eq. 2, we obtain an expression for \( f(p) \) in terms of \( F(p) \) and \( p \) that is conducive to numerical evaluation:

\[
f(p) = \frac{w_1(v - r)}{(p - r)^2 \sum_{i=2}^{S} i(i - 1)w_i[1 - F(p)]^{i-2}}
\]

The values of \( f(p) \) at the boundaries \( p^* \) and \( v \) are as follows:

\[
f(p^*) = \frac{\left( \sum_{i=1}^{S} iw_i \right)^2}{w_1(v - r) \left( \sum_{i=2}^{S} i(i - 1)w_i \right)} \quad \text{and} \quad f(v) = \frac{w_1}{2w_2(v - r)}
\]
Fig. 2(a) and 2(b) depict the PDFs in the prescribed model under varying distributions of buyer strategies — in particular, \( w_1 = 0.2 \) and \( w_2 + w_3 = 0.8 \) — when \( S = 5 \) and \( S = 20 \), respectively. In both figures, \( f(p) \) is bimodal when \( w_2 = 0 \), as is derived in Eq. 10. Most of the probability density is concentrated either just above \( p^* \), where sellers expect low margins but high volume, or just below \( v \), where they expect high margins but low volume. In addition, moving from \( S = 5 \) to \( S = 20 \), the boundary \( p^* \) decreases, and the area of the no-man’s land between these extremes diminishes. In contrast, when \( w_2, \omega_3 > 0 \), a peak appears in the distribution. If a seller does not charge the absolute lowest price when \( w_2 = 0 \), then it fails to obtain sales from any buyers of type \( S \). In the presence of buyers of type \( 2 \), however, sellers can obtain increased sales even when they are priced moderately. Thus, there is an incentive to price in this manner, as is depicted by the peak in the distribution. The case in which \( \omega_3 = 0 \); i.e., \( w_1 + w_2 = 1 \) is explored in more detail in the next section.

![PDFs for 5 Sellers](image1)

![PDFs for 20 Sellers](image2)

**Fig. 2.** PDFs for \( w_1 = 0.2 \) and \( w_2 + w_3 = 0.8 \).

Recall that the profit earned by each seller is \( \pi = (w_1/S)(v-r) \); this quantity is strictly positive so long as \( w_1 > 0 \). It is as though only buyers of type 1 are contributing to sellers’ profits, although the actual distribution of contributions from buyers of type 1 vs. buyers of type \( i > 1 \) is not as one-sided as it appears. In reality, buyers of type 1 are charged less than \( v \) on average, and buyers of type \( i > 1 \) are charged more than \( r \) on average, although total profits are equivalent to what they would be if the sellers practiced perfect price discrimination. In effect, buyers of type 1 exert negative externalities on buyers of type \( i > 1 \), by creating surplus profits for sellers.

### 3.2 Endogenous Buyer Decisions

Herefore in our analysis, we have assumed rational decision-making on the part of the sellers, but an exogenous distribution of buyer types. Given a vector of search costs \( c \), such that \( c_i \) denotes the cost of comparing the prices of \( i \) sellers, it
is also of interest to consider buyers as rational decision-makers, thereby giving rise to endogenous search behavior. As mentioned previously, rational buyers estimate the commodity’s price $\hat{p}_i$ that would be obtained by searching among $i$ sellers, and select the strategy $i^*$ that minimizes $\hat{p}_i + c_i$, provided that $\hat{p}_i + c_i \leq v_0$; otherwise, the buyer does not search and does not participate in the marketplace.

Before studying the decision-making processes taken by individual buyers, it is useful to analyze the distributions of prices paid by buyers of various types and their corresponding averages at equilibrium. Recall that a buyer who obtains $i$ price quotes pays the lowest of the $i$ prices observed. (At equilibrium, the sellers’ prices never exceed $v$ since $F(v) = 1$, so a buyer is always willing to pay the lowest price.) The cumulative distribution for the minimal values of $i$ independent samples taken from the distribution $f(p)$ is given by $Y_i(p) = 1 - [1 - F(p)]^i$. Differentiation with respect to $p$ yields the probability distribution: $y_i(p) = i f(p) [1 - F(p)]^{i-1}$. The average price for the distribution $y_i(p)$ can be expressed as follows:

$$\bar{p}_i = \int_{0}^{v} dp y_i(p) = v - \int_{0}^{v} dp Y_i(p) = p^* + \int_{0}^{1} dF \frac{(1 - F)^i}{i} \quad (11)$$

where the first equality is obtained via integration by parts, and the second depends on the observation that $dp/dF = [dF/dp]^{-1} = 1/f$. Combining Eqs. 3, 9, and 11 would lead to an integrand expressed purely in terms of $F$. Integration over the variable $F$ (as opposed to $p$) is advantageous because $F$ can be chosen to be equispaced, as standard numerical integration techniques require.

Fig. 3(a) depicts sample price distributions for buyers of various types: $y_1(p)$, $y_2(p)$, and $y_{20}(p)$, when $S = 20$ and $(w_1, w_2, w_{20}) = (0.2, 0.4, 0.4)$. The dashed lines represent the average prices $\bar{p}_i$ for $i \in \{1, 2, 20\}$ as computed by Eq. 11. The blue line labeled Search-1, which depicts the distribution $y_1(p)$, is identical to the green line labeled $w_2 = 0.4$ in Fig. 2(b), since $y_1(p) = f(p)$. In addition, the distributions shift toward lower values of $p$ for those buyers who base their buying decisions on information pertaining to more sellers.

Fig. 3(b) depicts the average buyer prices obtained by buyers of various types, when $w_1$ is fixed at 0.2 and $w_2 + w_{20} = 0.8$. The various values of $i$ (i.e., buyer types) are listed to the right of the curves. Notice that as $w_{20}$ increases, the average prices paid by those buyers who perform relatively few searches increases rather dramatically for larger values of $w_{20}$. This is because $w_1$ is fixed, which implies that the sellers’ profit surplus is similarly fixed; thus, as more and more buyers perform extensive searches, the average prices paid by those buyers decreases, which causes the average prices paid by the less diligent searchers to increase. The situation is slightly different for those buyers who perform larger searches but do not search the entire space of sellers: e.g., $i = 10$ and $i = 15$. These buyers initially reap the benefits of increasing the number of buyers of type 20, but eventually their average prices increase as well. Given a fixed portion of the population designated as buyers of type 1, Fig. 3(b) demonstrates that searching $S$ sellers is a superior buyer strategy to searching $1 < i < S$ sellers. Thus, there is value in performing price searches: shopbots offer added value in markets in which there exist buyers who shop at random.
Initially, we model buyer search costs following Burdett and Judd [2], who assume costs are linear in the number of searches; in particular, $c_i = c_1 + \delta(i-1)$, where $c_1, \delta > 0$ are, respectively, fixed and marginal costs of obtaining price quotes. Moreover, we assume buyers are rational decision-makers who strive to minimize overall expenditure, and who use $\bar{p}_i$ (as in Eq. 11) as an estimate of $\tilde{p}_i$. Thus, an optimal buyer strategy $i^*$ satisfies: $i^* \in \arg \min_{0 \leq i \leq S} \bar{p}_i + c_i$. At equilibrium, $0 < w_1 \leq 1$, since if $w_1 = 0$, then all buyers perform some degree of search, in which case all sellers charge the competitive price $r$ (see Eqs. 6 and 7), from which it follows that it is in fact not rational for buyers to search at all, leading to the contradiction that $w_1 = 1$. Now since the buyer cost function $\bar{p}_i + c_i$ is convex, it is minimized at either a single integer value $i^*$, or two consecutive integer values $i^*$ and $i^* + 1$. Thus, at equilibrium, either $w_1 = 1$, in which case all sellers charge the monopolistic price $v$, or $w_1 + w_2 = 1$ and the sellers’ prices are given by the distribution $f(p)$.

In the case where $w_1 + w_2 = 1$, by substituting Eq. 6 into Eq. 11, we obtain analytic expressions for the average prices seen by buyers of types 1 and 2:

$$p_1(w_2) = p^* + \left(1 - \frac{w_2}{w_1 + w_2}\right) \left[\log \left(\frac{1 + w_2}{1 - w_2}\right) - \frac{2w_2}{1 + w_2}\right] (v - r) \quad (12)$$

$$p_2(w_2) = p^* + \left(\frac{1 - w_2}{2w_2}\right) \left[2w_2 + (1 - w_2^2) \log \left(\frac{1 - w_2}{1 + w_2}\right)\right] (v - r) \quad (13)$$

Fig. 4(a) plots $p_1$ (i.e., Search–1) and $p_2$ (i.e., Search–2) as a function of $w_2$, given that $v = 1$ and $r = 0.5$. The curves decrease monotonically with $w_2$, reflecting the fact that prices decrease on average as the degree of search increases.

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4 This depends on the assumption that $c_1$ is sufficiently small such that $w_0 = 0$. Otherwise, the equilibria which arise are such that $w_1 = 1 - w_0$ or $w_1 + w_2 = 1 - w_0$. 
Fig. 4(b) plots the marginal benefit \( \beta \) of Search–2 over Search–1, which can be computed by subtracting Eq. 13 from 12:

\[
\beta(w_2) = \left(1 - \frac{w_2}{w_2}\right) \frac{1}{2w_2} \log \left(\frac{1 + w_2}{1 - w_2}\right) - 1 \quad (v - r)
\]

Suppose that all buyers are fully informed and rational, and therefore estimate expected marginal benefit accurately. Then, if buyers are free to choose between the strategies Search–1 and Search–2, the buyer population reaches equilibrium when the marginal benefit of a second price quote exactly balances its marginal cost. Fig. 4(b) graphically illustrates the situation in which the marginal cost \( \delta = c_2 - c_1 \) is finite and reasonably small; \( \delta \) is valued at 0.02 and depicted by the dotted line. There are two points of intersection on the marginal benefit and marginal cost curves in this diagram, representing 2 of the 3 equilibria that arise in this setting. Above the dotted line, benefit outweighs cost; thus, it is advantageous to search and there is momentum in the rightward direction. Below the dotted line, cost outweighs benefit, and it is therefore not desirable to search; hence, there is momentum in the leftward direction. Following the direction of the arrows, we observe that the filled-in circle that falls on the curve is a stable equilibrium, while the open circle represents an unstable equilibrium. The unstable equilibrium represents a boundary between two basins of attraction; a buyer population in which the initial value of \( w_2 \) is greater than this threshold will migrate towards the equilibrium near \( w_2 = 1 \), while one in which \( w_2 \) is initially smaller than this threshold will migrate towards \( w_1 = 1 \). In addition, there is a second stable equilibrium in the lower left-hand corner of the graph (indicated by a second filled-in circle) where \( w_1 = 1 \), the equilibrium price is the monopolistic price \( v \), and \( \lim_{w_2 \to 0} \beta(w_2) = \delta = 0 \).

![Average Buyer Prices](image1)

![Marginal Benefit of Search](image2)

(a) Average prices for buyers.

(b) Marginal benefit vs. \( w_2 \).

**Fig. 4.** Economy of buyers of type 1 and 2. Buyer valuation \( v = 1 \); seller cost \( r = 0.5 \).
Before closing this section, we note that expected buyer surplus per purchase, which we denote by $\sigma$, is defined as follows:

$$\sigma = \sum_i w_i (v - p_i - c_i)$$

(15)

This quantity is of particular interest in comparing overall social welfare between markets with and without shopbots.

4 Shopbot Experiments

In order to explore the likely effect of shopbots on market dynamics, we now consider two distinctive shopbot scenarios in turn, one in which search costs are nonlinear, and another in which some portion of the population does not have access to search mechanisms. The assumptions, the methodology, and the results presented herein differ substantially from traditional economic analyses.

4.1 Nonlinear search costs

A typical shopbot such as the one residing at www.DealPilot.com permits users to choose the number of sellers among whom to search. Since the service is free for buyers at present, and moreover, since the search is very fast—DealPilot searches prices at a few dozen book retailers within about 20 seconds — there is only a mild disincentive not to request a large number of price quotations. Thus, the effective search cost is only weakly dependent on the number of searches. One way to model weak dependence on the number of searches is via a nonlinear search cost schedule:

$$c_j = c_1 + \delta (j - 1)^\alpha,$$

(16)

where the exponent $\alpha$ is in the range $0 \leq \alpha \leq 1$. Note that $\alpha = 1$ yields a linear search cost model, while $\alpha = 0$ yields a search cost that is independent of the number of searches for $j > 1$.

Suppose that buyers periodically (but at random times) re-evaluate their search strategies and choose the strategy $j$ that minimizes $\hat{p}_j + c_j$, where $\hat{p}_j$ is their estimate of the average price they are likely to pay when using search strategy $j$. In determining $\hat{p}_j$, one possibility is that the buyers (or an agent acting on buyers’ behalf) use historical data on sellers’ prices to compute their estimates. We assume here, however, that the buyers are perfectly knowledgeable about the sellers’ marginal production cost $r$ and the current state of the strategy vector $\mathbf{w}$, and thus they integrate Eq. 11 numerically to compute $\hat{p}_j = \hat{p}_j$. As the buyers modify their strategies in this manner, we assume further that the sellers monitor $\mathbf{w}$, and instantaneously re-compute the symmetric price distribution $f(p)$ according to which they randomly choose their prices.

We can approximate this evolutionary process by a discrete time process in which, at each time step, a fraction $\eta$ of the buyer population is given the opportunity to switch to the optimal strategy. Then the strategy vector evolves
according to: $w_i(t + 1) = w_i(t) + \eta(\delta_{ij} - w_i(t))$, where $j$ is the strategy that minimizes $p_j + c_j$ and $\delta_{ij}$ represents the Kronecker delta function, equal to 1 when $i = j$ and 0 otherwise. Fig. 5(a) illustrates the evolution of the components of $\mathbf{w}$ in a 5-seller system when $w_1$ is completely endogenous ($[w_1] = 0$), the search costs are linear ($\alpha = 1, c_1 = 0.05$, and $\delta = 0.02$), and the value of $\eta$ is 0.002. Recall that according to Burdett and Judd [2], $\mathbf{w}$ must evolve toward an equilibrium consisting of a finite number of type 1 and type 2 buyers. Indeed, this does occur, but what is most interesting is the trajectory of the $\mathbf{w}$ on its route toward equilibrium.

![Graph](image)

**Fig. 5.** (a) Evolution of indicated components of buyer strategy vector $\mathbf{w}$ for 5 sellers, with linear search costs $c_i = 0.05 + 0.02(i - 1)$. Final equilibrium oscillates with small amplitude around theoretical solution involving a mixture of strategy types 1 and 2. (b) Evolution of indicated components of buyer strategy vector $\mathbf{w}$ for 5 sellers, with nonlinear search costs $c_i = 0.05 + 0.02(i - 1)^{0.85}$. Final equilibrium oscillates chaotically around a mixture of strategy types 1, 2, and 3.

Initially, $\mathbf{w}_0 = (0.2, 0.3, 0.0, 0.0, 0.5)$. In this situation, the favored strategy is type 3, and so $w_3$ begins to grow at the expense of $w_1$, $w_2$, and $w_5$. However, as $w_5$ diminishes, the total amount of search in the system diminishes, and $f(p)$ flattens and shifts in such a way that eventually the favored strategy shifts from 3 to 2. Thereafter, $w_2$ grows at the expense of $w_3$ and the other components. In this simulation, near but imperfect equilibrium is achieved; due to the finite size of $\eta$ (equal to 0.002), there are small oscillations in $w_2$ around an average value that is close to the theoretical value of 0.9641721. This value can be derived by identifying the value of $w_2$ corresponding to $\delta = 0.02$ in Fig. 4(b). In Fig. 4(b), there is a second value of $w_2$ satisfying $\delta = 0.02$, near $w_2 = 0.137564$. However, this is the unstable equilibrium, and as discussed in the previous section it marks the boundary between two basins of attraction, one in which the final equilibrium is $(w_1, w_2) = (0.0358279, 0.9641721)$, and the other in which $(w_1, w_2) = (1, 0)$.

The derivation of an equilibrium in which only type 1 and type 2 strategies could co-exist was founded on the assumption that search costs are linear in the amount of search. In order to investigate the effect of nonlinear search costs that
grow only weakly with the amount of search, we run the same experiment, in which all parameters are identical except for the exponent \( \alpha \), which is reduced from 1.0 to 0.25. Fig. 5(b) depicts the result. Interestingly, in this case the system evolves to an equilibrium in which types 1, 2 and 3 co-exist: \( w \) oscillates around the value (0.0217, 0.5357, 0.4426, 0.0000, 0.0000) in a way that appears to be chaotic; it remains to conduct further tests of this phenomenon. While these oscillations are an artifact of the finite size of \( \eta \), and would likely disappear in the limit \( \eta \rightarrow 0 \), they hint at the fact that the system might still undergo large-scale nonlinear and possibly chaotic oscillations if the buyers were to revise their strategies synchronously rather than asynchronously.

4.2 Lower limit on \( w_1 \)

Today’s shopbots are used by only a small fraction of shoppers. This is due at least in part to the fact that many potential users are unaware of the existence of shopbots, and others do not know where to find them or how to use them. One way of modeling buyers who do not use shopbots is to assume that such uninformed users are buyers of type 1, for which they incur only fixed cost \( c_1 \). This establishes a lower limit on the fraction \( w_1 \), which we denote \( [w_1] \). In particular, \( [w_1] \) represents the fraction of uninformed buyers who guarantee the sellers a strictly positive profit surplus.

In order to explore the outcome of some proportion of buyers failing to adopt low-cost search methods (perhaps due to ignorance about shopbots’ existence or about how to use them), we now impose a lower limit on \( w_1 \), denoted \( [w_1] \). Fig. 6(a) depicts the result of imposing \( [w_1] = 0.04 \), with linear search costs \( c_i = 0.05 + 0.005(i - 1) \). Allowing the system to evolve from initial strategy vector \( w_0 = (0.04, 0.20, 0.00, 0.00, 0.76) \), the system reaches an equilibrium in which only types 1 and 4 co-exist, with \( w_1 = 0.04 \) and \( w_4 = 0.96 \), rather than types 1 and 2 as was the case in the traditional economic setting that was analyzed in Sec. 3.2.

In numerous experiments with \( w_1 \) bounded below and linear search costs, we have observed that the final equilibrium always consists of a mixture of buyer types 1 and \( i \), where \( i \) is not necessarily 2, as it must be when \( w_1 \) is determined in an entirely endogenous fashion. The strategy \( i \) depends on the values of \( [w_1] \) and \( \delta \). Table 1 illustrates the dependence of the strategy \( i \) that mixes with strategy 1 upon \( [w_1] \) and the incremental cost \( \delta \). Higher values of \( [w_1] \) lead to higher equilibrium strategies \( i \) (more extensive search) while higher incremental costs \( \delta \) lead to lower equilibrium strategies \( i \) (less extensive search). For the table entries \( ([w_1], \delta) = (0.04, 0.005) \) and \( ([w_1], \delta) = (0.20, 0.020) \), multiple equilibria are obtained. In these cases, the initial setting of the strategy vector determines which equilibrium obtains.

The effect of initial conditions on equilibrium selection in the case \( ([w_1], \delta) = (0.04, 0.005) \) is illustrated in Fig. 6(b). Four equilibria are possible, all of the form \( w_1 + w_i = 1 \), for \( i = 2, 3, 4, 5 \). The set of initial conditions leading to equilibrium \( i \) — its “basin of attraction” — forms a contiguous, smoothly bounded region, a two-dimensional cross-section of which is depicted in Fig. 6(b).
<table>
<thead>
<tr>
<th>$[w_1]$</th>
<th>$\delta = 0.001$</th>
<th>$\delta = 0.005$</th>
<th>$\delta = 0.020$</th>
</tr>
</thead>
<tbody>
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<td>2</td>
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<td>5</td>
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<td>2</td>
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<tr>
<td>0.20</td>
<td>5</td>
<td>5</td>
<td>2-3</td>
</tr>
</tbody>
</table>

Table 1. Search strategy or strategies that co-exist with type 1 search strategy, as a function of $[w_1]$ and incremental cost $\delta$.

Fig. 6. (a) Evolution of indicated components of buyer strategy vector $w$ for 5 sellers, with linear search costs $c_i = 0.05 + 0.02(i - 1)$ and $[w_1] = 0.04$. Starting from $w_0$ indicated in the text, $w$ evolves towards equilibrium with only types 1 and 4 present. (b) Two-dimensional cross-section of basin of attraction for $([w_1], \delta) = (0.04, 0.005)$.

5 Shopbots as Economic Agents

A truly unique aspect of shopbots is that they possess the (as yet unrealized) potential to act as economic agents themselves — that is, they could charge buyers directly for providing a pricing information service. Thus, search costs need not be merely an exogenously determined constant — they could be set strategically by shopbots in an effort to maximize their own profits. In this section, we suppose that a shopbot charges a price $c_i$ for $i$ randomly chosen price quotes, and we examine how it can judiciously set its price schedule $c_i$ so as to maximize profit. We assume the shopbot has no variable production costs.

For a start, assume that the shopbot is the only means by which buyers can obtain price quotes. Then the shopbot can easily extract the entire surplus from the market by setting $c_1 = c_2 = v - r$. According to this cost schedule, buyers do not pay an additional amount for a second quote; thus, they all request two quotes. In particular, $(w_1, w_2) = (0, 1)$, and the sellers charge the marginal cost $r$, obtaining no surplus at all. The sum of the expected cost of the item and the search cost is precisely the buyer valuation $v$, so the buyers buy the item, also receiving no surplus. All of the surplus $(v - r)$ goes to the shopbot!

It is more natural to suppose that the buyers have an alternative mechanism by which they can discover prices, such as manual search. In this scenario, all of the buyers can weigh the costs and benefits of using the shopbot against those of
using the alternative search mechanism to decide which mechanism to use and how much search to perform or request. In this case, the shopbot’s profit $\pi_{sh}(t)$ is given by:

$$\pi_{sh}(t) = \sum_{i=1}^{\tilde{S}} w_i (c, c_{\text{alt}}, t) c_i \Theta(c_{\text{alt}, i} - c_i) \Theta(v - (p_i + c_i))$$

(17)

where $\Theta(x)$ represents the step function, equal to 1 for $x > 0$ and 0 otherwise. The two step functions represent the fact that, in order to capture the $Search$ segment of the market, the shopbot must both undercut the cost of using the alternate search mechanism and must price low enough so that the search cost plus the expected item price does not exceed the buyer’s valuation.

Recall from the previous section that since buyers’ strategy choices depend on other buyers’ choices, strategy vector $w$ may have a complex time dependency. We can obtain some insights into the optimal pricing of prices, however, by considering a simple example that is analytically tractable.

Suppose that the cost of the alternative search mechanism is linear in the number of price quotes: $c_{\text{alt}}(i) = ic'(v-r)$. Furthermore, assume that the shopbot restricts itself to offering only 1 or 2 quotes. Then its task is to set the values of $c_1$ and $\delta = c_2 - c_1$ so as to maximize its expected profit. Then, as shown in Sec. 3.2, rational, fully-informed buyers will evolve to an equilibrium $w_2$ in which the marginal benefit and cost of a second quote is balanced. Suppose that the shopbot takes a long-term view, so that it does not factor transients into its calculations, but only seeks to set a price structure $c$ to maximize Eq. 17 with $w$ set to its asymptotic equilibrium value. Finally, without loss of generality, we can set $v = 1$ and $r = 0$; generalized expressions for all derived quantities can be obtained by a simple rescaling.

Using the fact that, at equilibrium, the marginal benefit $\beta(w_2)$ equals the marginal cost $\delta$, the shopbot’s profit can be written as follows:

$$\pi_{sh}(t) = w_1 c_1 + w_2 c_2 = c_1 + w_2 \delta = c_1 + w_2 \beta(w_2).$$

(18)

subject to the conditions:

$$c_1 < c'$$

(19)

$$c_2 < 2c'$$

$$c_1 < v - \bar{p}_1$$

$$c_2 < v - \bar{p}_2$$

Taken together, the first condition in Eq. 19 and the rightmost expression for the shopbot’s profit in Eq. 18 suggest that $c_1$ should always be chosen to just undercut $c'$. The fourth condition may be eliminated because it is redundant with the third, given that the marginal benefit and marginal cost of the second quote are equal when $w$ is in equilibrium.

Temporarily ignoring the second and third conditions, it is apparent from Eq. 18 that the shopbot’s profit is maximized when $w_2 \beta(w_2)$ is maximized.
Using Eq. 14 and solving numerically for the optimal value of $w_2$, we find that it occurs at $w^*_2 = 0.781796$. If $w_2$ is less than this value, the expected item prices $\bar{p}_1$ and $\bar{p}_2$ increase and so does their differential $\beta$, but the increase in $\beta$ fails to compensate for the reduction in $w_2$. On the other hand, if $w_2$ is greater than $w^*_2$, the expected item prices and the price differential $\beta$ decrease more than would be compensated by the increase in $w_2$.

In order for the shopbot to encourage the buyer population to evolve to $w_2$, it should set the price differential $\delta$ to the corresponding optimal $\delta^* = \beta(w^*_2) = 0.095741$. This strategy, however, is not guaranteed to be successful, because (as illustrated in Fig. 4(b) for $\delta = 0.02$) there are two values of $w_2$ that correspond to $\delta^* = 0.095741$: the desired value $w^*_2 = 0.781796$ and an additional unstable solution at $w_2 = 0.465602$. As discussed in Sec. 3.2, the system will evolve to a state in which $w_2 = 0$ if the initial value of $w_2$ is less than 0.465602. To cope with low initial values of $w_2$, the shopbot could use a more sophisticated strategy. It could deliberately charge a very small $\delta$ initially, such that the unstable equilibrium for $w_2$ is less than the initial value of $w_2$. This would cause $w_2$ to increase, whereupon the shopbot could gradually raise $\delta$ up to the desired value of $\delta^*$.

Now consider the second condition in Eq. 19. It will be violated if $c_2 \geq 2c'$, which occurs when $c' < \delta^*$. In this regime, the shopbot cannot charge $\delta^*$ for the second quote because buyers wishing to purchase two quotes would choose the cheaper alternate search method. Instead, the shopbot must undercut the alternate search method, which it may do by setting $\delta = c'$.

Finally, consider the third condition. Substitution of Eqs. 12 and 7 yields the constraint $c_1 < \phi(w_2)$, where

$$\phi(w_2) \equiv 1 - \left[ \frac{1 - w_2}{2w_2} \right] \log \left( \frac{1 + w_2}{1 - w_2} \right)$$

This constraint comes into play when $c' > \phi(w^*_2) = 0.706946$. In this regime, $c'$ is large enough so that, with $\delta = \delta^*$, the expected item cost plus the search cost would exceed the buyer’s valuation. This would cause the buyers to opt out of the market, resulting in no profit for the shopbot. In order to decrease the total cost to the buyer, the shopbot can still set $c_1$ to just undercut $c'$, but it must reduce the overall price to the buyer by manipulating $\delta$ so as to increase $w_2$ above $w^*_2 = 0.781796$, which in turn decreases $\bar{p}_1$ and $\bar{p}_2$. The shopbot can achieve this by reducing $\delta$ below $\delta^*$. In this case, the optimal value of $w_2$ is determined by inverting $c' = \phi(w_2)$, or $w_2 = \phi^{-1}(c')$, and $\delta = \beta(w_2) = \beta(\phi^{-1}(c'))$. It can be shown that, in this regime, $\pi_{sh}$ is precisely equal to $w_2$.

In addition to these three distinct ranges of $c'$, there is a fourth scenario in which $c' > 1$. In this case, the buyers cannot afford the alternate search mechanism. This is tantamount to the shopbot being the only search mechanism, and as discussed earlier, the shopbot extracts all of the market surplus.

The analytic results for $c_1$, $\delta - c_2 - c_1$, $w_2$, and $\pi_{sh}$ in these four different ranges of $c'$ are summarized in Table 5 and illustrated in Fig. 7. In Table 5 and Fig. 7, $(v - r)$ is normalized to 1; the result for general $(v - r)$ can be obtained by multiplying all search cost parameters (e.g., $c'$, $c_1$, and $\delta$) by this quantity.
Table 2. Optimal shopbot prices $c_1$ and $c_2$, Strategy-2 population $w_2$, and shopbot profit $\pi_{sh}$ as a function of the alternate search cost $c'$. Special values $\delta' = 0.095741$, $w_2^* = 0.781796$ and $\phi(w_2^*) = 0.706946$ are defined in the text.

Fig. 7. Optimal shopbot parameters as a function of alternative search cost $c'$, with $v - r$ normalized to 1. a) Shopbot prices $c_1$ and $c_2$ as a function of $c'$. b) Population of Strategy-2 buyers $w_2$, shopbot profit $\pi_{sh}$, total seller profit $\pi$, and buyer surplus $\sigma$.

Figure 8(a) displays two additional quantities of interest: the total seller profit (summed over all sellers), which by Eq. 8 is simply $w_1 = 1 - w_2$, and the buyer surplus $\sigma$, which in this case is as follows:

$$\sigma = \sum_i w_i (v - \bar{v}_i) - \pi_{sh}$$  \hspace{1cm} (21)

$$= \phi(w_2) + w_2\beta(w_2) - \pi_{sh}$$

$$= w_2 - \pi_{sh}$$

Note therefore that the total social welfare is simply $S\pi + \sigma + \pi_{sh} = 1$. For general $v$ and $r$, these quantities can be rescaled to yield a total social welfare of $v - r$. This is a consequence of the assumption that all buyers have equal valuations $v$, and the fact that the shopbots are motivated to manipulate the market to ensure that all buyers purchase the item.
Fig. 8. Buyer surplus \( \sigma \) and total seller profit \( S \pi \) as a function of alternate search cost \( c' \) under two different assumptions: a) shopbot sets \( c_1 \) and \( c_2 \) to maximize its own profit, and b) no shopbot present.

For comparison, Fig. 8(b) depicts the buyer surplus and seller profit in the case where there is no shopbot, i.e., the buyers simply pay \( c' \) for one quote and \( 2c' \) for two quotes. There are two regimes. Recall that if \( \delta = c' \) is sufficiently small, there are two solutions to \( \beta(w_2) = \delta \), one of which is stable. Strategy-1 and Strategy-2 buyers can co-exist at the non-trivial stable equilibrium defined by \( \beta(w_2) = c' \) provided that \( c' < 0.103872 \), the maximal value attained by \( \beta(w_2) \) (this occurs at \( w_2 = 0.634816 \)). Again, this assumes that the initial conditions are such that the population will evolve towards the non-trivial equilibrium rather than the one at \( w_2 = 0 \). In the range \( c' < 0.103872 \), the total seller profit is simply \( 1 - w_2 = 1 - \beta^{-1}(c') \), and the buyer surplus is \( \sigma = (1 - c')w_2 - c' = (1 - c')\beta^{-1}(c') - c' \). However, if \( c' \) exceeds the threshold 0.103872, then there is no solution such that \( \beta(w_2) = c' \), and the system will evolve to the only stable equilibrium: the trivial one at \( w_2 = 0 \). Since none of the buyers compare prices, the sellers are free to behave as monopolists. All sellers set an item price of \( v - c' \), which is the maximum that the buyers will pay, given that they must pay \( c' \) to discover the sellers’ price. Thus the total seller profit is \( v - c' \) and the buyer surplus is exactly 0. These curves are plotted in Fig. 8(b).

When \( c' \) is small, the buyer surplus is relatively high, and is completely unaffected by the presence or absence of the shopbot. This is because the shopbot is forced to just undercut the alternate search mechanism on both the single-quote and double-quote prices, so the buyer sees no difference in the search cost. However, the behaviors are quite different in an intermediate range of \( c' \). Without the shopbot, the buyer surplus drops to zero above \( c' = 0.103872 \). With the shopbot, however, the surplus drops but remains positive all the way up to
$c' = 0.706946$. For larger $c'$, the buyer surplus is zero in both cases. Overall, despite the fact that the shopbot is seeking only to maximize its own profit, it still provides a significant benefit to buyers by extending the range of $c'$ for which they can experience a positive surplus.

In the foregoing analysis, we have assumed that the shopbot only provides one or two quotes. Of course, this was done purely for reasons of tractability — there is no practical reason for a shopbot to constrain itself in such a way. If freed from this constraint, how might a shopbot maximize its profits? One plausible method would be to use a numerical optimization technique in which each candidate price schedule is evaluated by simulating the evolution of the buyers’ strategies until an equilibrium or approximate equilibrium is reached (just as was done in Sec. 4). However, even this computationally intensive approach is likely to be insufficient, given our observation that the shopbot may need to employ a dynamic price schedule to encourage the buyers to reach the desired final equilibrium.

One heuristic for setting the price schedule dynamically is to set $c_1 = c' - \epsilon$ and then to set the remaining prices so that $\tilde{p}_i + c_i$ is constant for all $i$. In other words, at any moment in time, the shopbot sets the marginal price of quote $i$ ($c_i$) equal to the marginal benefit of that quote to a buyer, given the current setting of $w$. Then a fraction $\eta$ of buyers switch their strategies. $w$ is updated to reflect this, and the cycle begins afresh in the next time step. Note that the algorithm causes buyers to be indifferent as to which search strategy they choose. This heuristic strongly encourages $c$ and $w$ to settle to an equilibrium. In fact, it can be over-aggressive, causing the shopbot to settle upon a price structure that is not quite optimal, but in practice it seems to yield reasonably good solutions.

Figure 9 depicts a simulation in which, initially, all parameters are exactly as in Fig. 5(a). Then, at time $t = 1000$ (and at intervals of every 100 time steps thereafter) the shopbot pricing heuristic is put into effect. The cost of the alternate search mechanism is taken to be $c' = 0.25$ per quote. Immediately, the $c_i$ shift from a linear dependence on the search strategy $i$ to a highly nonlinear one, and the shopbot prices $c$ and buyer strategies $w$ continue to co-evolve for a while until they finally reach an equilibrium in which the $c_i$ approximate the nonlinear form assumed in Eq. 16, with $\alpha \approx 0.2$. In effect, the shopbot is selling the set of all five price quotes as a bundle, encouraging almost all buyers to switch to purchasing the full bundle rather than just purchasing two quotes, as they preferred to do with linear costs. This switchover to a strong preference for $Search-5$ is reflected in Fig. 9(a). The shopbot’s profit increases dramatically as soon as the heuristic is put into effect, and increases further to roughly 0.359 by the end of the simulation run at time $t = 10000$. Although this is not known to be an optimal solution, it still compares favorably with the maximal profit of 0.287 that could have been obtained by the shopbot had it only offered one or two quotes. (The theoretical optimum for at most two quotes is computed by multiplying the results in Table 5 by $(v - r) = 0.5$..)
Fig. 9. Heuristic pricing of prices by a monopolist shopbot. a) $w(t)$. b) $c(t)$. c) $\pi(t)$
6 Related Work

The study of the economics of information was launched in the seminal work of Stigler [17] in 1961. In this paper, Stigler cites several examples of observed price dispersion, which he attributes to the costly search procedures that consumers face. Consequently, he notes the utility of trade journals and organizations that specialize in the collection and dissemination of product information, such as Consumer Reports and, of course, shopbots. Stigler reminds us that in medieval times, localized marketplaces thrived in spite of heavy taxes that were levied on merchants, demonstrating how worthwhile it was for sellers to participate in localized markets rather than search for buyers individually. Similarly, shopbots today serve as local marketplaces in the global information superhighway, and accordingly, we find sellers sponsoring shopbots and paying commissions on sales, as they essentially pay for the right to participate in the shopbot marketplace. This strategy encapsulates the current business model of Internet sites such as DealPilot.com that design shopbots.

Since Stigler, many economists have developed and analyzed formal models that attempt to explain the phenomenon of price dispersion. We have already mentioned the work of Varian [19] and Burdett and Judd [2]. The former study is a special case of our model of shopbot economics in the case of two types of buyers, namely type 1 and type 2, existing in fixed proportions. The latter authors, on the other hand, consider independent buyer decisions among a set of search rules of fixed sample size. In this paper, we integrate these two approaches by specifying a fixed minimal proportion of buyers of type 1 while allowing all other buyers to choose their sample sizes. Moreover, we extend these ideas with the notion that shopbots themselves may well price for the information services they provide, acting as economic agents in their own right; this creates an additional strategic variable, namely the cost of search, as determined by shopbots attempting to maximize their profitability.

In the absence of shopbots and strategic search costs, a model of sellers’ price adjustment was studied in Diamond [5], in which a somewhat paradoxical outcome arises: for any positive search costs, no consumers search and all sellers charge the monopolistic price \( p \). More recent work on the dynamics of price-setting includes the evolutionary approach of Hopkins and Seymour [11], where it is argued that the mixed strategy game-theoretic equilibrium is dynamically unstable. This raises legitimate concerns about the validity of our assumption that \( f(p) \) describes sellers’ behavior. On the other hand, since we also assume that sellers are represented by software agents, it is in fact plausible to suppose that these agents would price in this manner, as the computation of \( f(p) \) is a simple and well-specified algorithm. Another recent work of relevance which considers the potential impact of reduced buyer search costs on the electronic marketplace is Bakos [1]. This model is in some sense more general than ours in

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5 We mention only a handful of papers that make up this large body of literature, but refer the reader to the bibliography included in Hopkins and Seymour [11] for additional sources.
that it allows for product differentiation, but it does not allow for varying types among buyers. It remains to incorporate features of product differentiation with the buyer search strategies of the present model.

7 Conclusions and Future Work

Our desire to explore the economic impact of shopbots in obtaining price and product information has led us to a model that is similar in spirit to several that have previously been investigated by economists interested in understanding the phenomenon of price dispersion. Our goals, however, are prescriptive, rather than descriptive, leading us to consider somewhat different causes and effects than are typical of price dispersion studies. Ultimately, we are interested in designing economically-motivated software agents, as well as an infrastructure that will support their interactions; thus, we have emphasized the constructive computation of price distributions and averages, rather than merely providing classical proofs of existence and other properties of equilibria.

Arguing that nonlinear search cost schedules are likely to exist naturally, or might even be adopted intentionally by shopbots, we studied their effect within the context of our model; our findings reveal that nonlinear search costs can lead to more complicated mixtures of buyer strategies and more extensive search than occur with linear costs. Another practical assumption, namely the existence of a positive number of uninformed buyers who do not use search mechanisms, can lead to similar outcomes. Taking evolutionary dynamics of buyer strategies into account, we found that the final equilibrium strategy vector depends on its initial value, and the route toward equilibrium can be surprisingly complicated.

Placing ourselves in the role of shopbot designers, we explored the strategic pricing of price information. Through modeling, analysis, and simulation, we validated our earlier assumption that nonlinear functions are reflective of search costs in electronic marketplaces by showing that nonlinear cost schedules come about as the natural consequence of economic incentives on the part of shopbots. Even in the face of competition from more costly search mechanisms, shopbots can wield a good deal of control over markets; specifically, they can manipulate prices so as to extract a large fraction of a market surplus. We demonstrated how buyers can benefit from this self-interested price manipulation. Shopbots' power would of course be diminished considerably if they were to compete amongst themselves. A study of coupled markets in which shopbots compete as providers of price and product information services to buyers or buyer agents would be fascinating.

In closing, we briefly mention two promising areas for future work. Firstly, combining the evolutionary dynamics of buyers with more realistic models of seller pricing behavior such as those described in [8,12] seems likely to generate interesting, complex dynamics, and moreover, would of practical significance. Secondly, since shopbots are starting to provide additional information about product attributes, it would also be of interest to analyze and simulate a model that accounts for both horizontal [1] and vertical differentiation.
A Appendix

In this appendix, it is shown that there is no pure strategy Nash equilibrium in our model of shopbot economics whenever \( 0 < w_1 < 1 \); if \( w_1 = 1 \), then the unique Nash equilibrium is such that all sellers charge the monopoly price \( v \); if \( w_1 = 0 \), then the unique Nash equilibrium is such that all sellers charge price the competitive price \( r \) (see [8,9]). The proof proceeds by contradiction: we assume the existence of a pure strategy Nash equilibrium and we derive the unique form of such an equilibrium, but we argue that a strategy profile of this form is not in fact an equilibrium.

Assuming the existence of a pure strategy Nash equilibrium, we rederive Eq. 2 of Sec. 3.1, which describes the probability \( h_s(p, w) \) that buyers choose seller \( s \). As in Sec. 3.1,

\[
h_s(p, w) = \sum_{i=0}^{S} w_i h_{s,i}(p)
\]

(22)

where \( h_{s,i}(p) \) denotes the demand for seller \( s \) by buyers of type \( i \). Since we are now assuming pure rather than mixed strategies, it is convenient to define the following functions in deriving \( h_{s,i}(p) \), a quantity whose probabilistic analog was previously expressed in terms of cumulative distribution functions:

- \( \mu_s(p) \) is the number of sellers charging a higher price than \( s \);
- \( \tau_s(p) \) is the number of sellers charging the same price as \( s \), excluding \( s \) itself;
- \( \lambda_s(p) \) is the number of sellers charging a lower price than \( s \).

The quantity \( h_{s,i}(p) \) is is the product of the probability \( x_s \) that \( s \) is among \( i \) potential sellers selected at random, and the conditional probability \( y_s \) that \( s \) is either the lowest-priced seller or the lucky one in a set of lowest-priced sellers, given a set of \( i \) potential sellers that includes \( s \). The probability \( x_s \) that \( s \) is one of a set of \( i \) potential sellers selected at random is simply \( x_s = \binom{S-1}{i-1} / \binom{S}{i} \).

The conditional probability \( y_s \) is a function of \( \lambda_s(p), \tau_s(p), \) and \( \mu_s(p) \). For convenience, fix seller \( s \), and abbreviate \( \nu \equiv \nu_s(p) \), for \( \nu \in \{\lambda, \mu, \tau\} \). Given \( \tau \) sellers charging the same price as seller \( s \), the probability \( y_s \) that \( s \) is one of the lowest-priced sellers among \( i \) potential sellers, and moreover, that \( s \) is randomly selected from among \( 0 \leq t \leq \tau \) such lowest-priced sellers, is given by:

\[
y_s(\lambda, \tau, \mu) = \sum_{t=0}^{\tau} \frac{1}{t+1} x_s(\lambda, \tau, \mu, t)
\]

(23)

where \( x_s(\cdot, t) \) denotes the probability that \( s \) is one of the \( t \) lowest-priced sellers of \( i \) potential sellers. The quantity \( x_s(\cdot, t) \) is computed by determining the number of ways in which to arrange \( i-1 \) of the remaining \( S-1 \) sellers, exclusive of seller \( s \), such that \( t \) sellers charge the same price as \( s \), but no seller charges a price less than \( s \), divided by the total number of ways in which to choose \( i-1 \) other potential sellers from the remaining \( S-1 \); i.e.,

\[
x_s(\lambda, \tau, \mu, t) = \binom{i-1}{t} \binom{\mu}{i-1-t} \binom{S-1-i}{\tau-i}
\]

(24)
Combining the expressions for \( x_s, y_s, \) and \( z_s \) and simplifying, we arrive at the following:

\[
h_s(x) = \left( \frac{S}{i} \right)^{-1} \frac{1}{\tau + 1} \sum_{t=0}^{\tau} \left( \frac{\tau + 1}{t + 1} \right) (i - (t + 1))
\]  

(25)

Finally, as in Sec. 3.1, let \( \rho B = 1 \) and \( g(p) = \Theta(v - p) \), which yields the sellers’ profit function \( \pi = (p - r)h_s(p, w) \) with \( h_s(p, w) \) defined by Eqs. 22 and 25.

We now proceed to derive the unique form of a possible pure strategy Nash equilibrium. Suppose that the sellers are ordered \( s_1, \ldots, s_j, \ldots, s_S \) such that the indices \( j < j + 1 \) whenever equilibrium prices \( p^*_j \leq p^*_{j+1} \). First, note that equilibrium prices \( p^*_j \in (r, v) \), since \( p_j < r \) yields strictly negative profits, while \( p_j = r \) and \( p_j > v \) yield zero profits, but \( p_j \in (r, v] \) yields strictly positive profits. The following observation describes the form of a pure strategy Nash equilibrium whenever \( 0 < w_1 < 1: \) at equilibrium, no two sellers charge identical prices.

Suppose on the contrary, that two distinct sellers offer equivalent prices; i.e., \( r < p^*_1 < \ldots < p^*_j = p^*_{j+1} < \ldots < p^*_S \leq v \). In this case, seller \( j \) stands to gain by undercutting seller \( j + 1 \) by \( \epsilon \), implying that \( p^*_j \) is not in fact an equilibrium price. In particular,

\[
\pi_j(p^*_j - \epsilon, p^*_{j+1}) = \sum_{i=0}^{S} w_i \left( \frac{\mu}{i} \right)^{-1} \left( \frac{\mu + 1}{i + 1} \right) (p_j - \epsilon - r) \\
> \sum_{i=0}^{S} w_i \left( \frac{\mu}{i} \right)^{-1} \left[ \frac{1}{i + 1} \right] (p_j - r) = \pi_j(p^*_j, p^*_{j+1})
\]

for sufficiently small values of \( \epsilon \), namely whenever

\[
\epsilon < \frac{i - 1}{2S} (p_j - r)
\]

(26)

Note that \( \epsilon > 0 \) just in case \( i > 1 \). In other words, it pays for sellers to undercut only so long as the market contains buyers of type \( i > 1 \); otherwise, \( \epsilon < 0 \), which implies that sellers stand to gain by increasing prices.

Further, we also observe that seller \( S \) charges price \( v \) at equilibrium, since for all \( p^*_S < v \), \( \pi_S(v, p^*_{S-1}) = \frac{1}{2} w_A(v - r) > \frac{1}{2} w_A(p_S - r) = \pi_S(p_S, p^*_{S-1}) \). Therefore, the relevant price vector consists of \( S \) distinct prices ordered such that \( r < p^*_1 < \ldots < p^*_S \).

The price vector \( (p^*_1, \ldots, p^*_j, \ldots, p^*_S) \), however, is not a Nash equilibrium. While \( p^*_S = v \) is in fact an optimal response to \( p^*_{S-1} \), since the profits of seller \( S \) are maximized at \( v \) given that there exists lower priced sellers \( 1, \ldots, S-1 \), price \( p^*_j \) is not an optimal response to \( p^*_{j+1} \). On the contrary, for all sellers \( 1 \leq j \leq S-1 \), seller \( j \) has incentive to deviate, since

\[
\pi_j(p^*_{j+1} - \epsilon, p^*_{j+1}) = \sum_{i=0}^{S} w_i \left( \frac{\mu}{i} \right)^{-1} \left( \frac{\mu + 1}{i + 1} \right) (p_j - \epsilon - r) \\
> \sum_{i=0}^{S} w_i \left( \frac{\mu}{i} \right)^{-1} \left( \frac{\mu + 1}{i + 1} \right) (p_j - r) = \pi_j(p^*_j, p^*_{j+1})
\]

It follows that there is no pure strategy Nash equilibrium in the proposed shopbot model, whenever \( 0 < w_1 < 1 \).
References